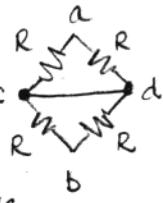


1. (a) Equivalent to

Resistance between a and b is unaffected by short from c to d because all R's the same.



$$\text{It is } (R//R) + (R//R) = R/2 + R/2 = R$$

$$\text{where } R = \boxed{\frac{I}{A}}$$

$$(b) \quad \xi_c - \xi_b = \xi_{in} \frac{1/iwC}{iwL + 1/iwC}$$

$$\xi_d - \xi_b = \xi_{in} \frac{iwL}{iwL + 1/iwC}$$

$$\begin{aligned} \xi_{out} &= \xi_c - \xi_d = \frac{1/iwC - iwL}{1/iwC + iwL} \\ &= \frac{1 + w^2 LC}{1 - w^2 LC} \quad \boxed{\neq 0 \text{ } \forall w} \end{aligned}$$

$$2. \quad \pi^+ \rightarrow \mu^+ + \nu$$

Assume μ^+ along \hat{x} :

$$(m_\pi c, 0, 0, 0) \rightarrow (\sqrt{p^2 + m_\mu^2 c^2}, p, 0, 0) + (p, -p, 0, 0)$$

E consv:

$$m_\pi c - p = \sqrt{p^2 + m_\mu^2 c^2}$$

$$m_\pi^2 c^2 - 2m_\pi c p + p^2 = p^2 + m_\mu^2 c^2$$

$$p = \frac{(m_\pi^2 - m_\mu^2)c^2}{2m_\pi c} = \frac{m_\pi c}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$$

$$\frac{E_\mu}{c} = \sqrt{p^2 + m_\mu^2 c^2}$$

$$\text{But } p = \gamma_\mu \beta_\mu m c, \quad E_\mu = \gamma_\mu m c^2$$

$$\therefore \beta_\mu^2 = \frac{p^2}{E_\mu^2/c^2} = \frac{p^2}{p^2 + m_\mu^2 c^2} = \frac{1}{1 + \frac{m_\mu^2 c^2}{p^2}}$$

$$\beta_\mu^2 = \frac{1}{1 + \frac{m_\mu^2 c^2}{\frac{m_\pi^2 c^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}}$$

Define $r = m_\mu/m_\pi$ Then

$$\beta_\mu^2 = \frac{1}{1 + \frac{4r^2}{(1-r^2)^2}} = \frac{(1-r^2)^2}{1-2r^2+r^4+4r^2}$$

$$\beta_\mu^2 = \frac{(1-r^2)^2}{(1+r^2)^2}, \quad \beta_\mu = \frac{1-r^2}{1+r^2}$$

$$\beta_\mu = \frac{1-9/16}{1+9/16} = \boxed{\frac{7}{25}}$$

$$3. (a) \quad \vec{A} = \frac{\alpha}{2} \hat{z} (y^2 - x^2)$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \hat{x} \frac{\partial A_z}{\partial y} - \hat{y} \frac{\partial A_z}{\partial x} = \alpha (\hat{x}y + \hat{y}x)$$

$$\vec{F} = -\frac{e}{c} \vec{v} \times \vec{B} = -e\alpha (0, 0, \frac{v}{c}) \times (y, x=0, 0)$$

$$\vec{F} = \boxed{-e\alpha \frac{v}{c} y \hat{y}}$$

$$\begin{aligned} (b) \quad \frac{4\pi}{c} \vec{J} &= \vec{\nabla} \times \vec{B} = \hat{z} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \\ &\quad + \hat{x} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) + \hat{y} \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) \\ &= \boxed{0} \end{aligned}$$

$$(c) \quad \frac{4\pi}{c} \vec{J} = A_0 \vec{\nabla} \times (\hat{x}y - \hat{y}x) = A_0 \hat{z} (-1 - 1)$$

(using results of part (b))

$$\vec{J} = \boxed{-\hat{z} \frac{c}{2\pi} A_0}$$

$$4. (a) \quad \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$(b) \quad \vec{\nabla} \times \vec{H} = \frac{4\pi}{c} \vec{J}_{\text{free}}^0 + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}$$

$$(c) \quad \vec{D} = \epsilon \vec{E}, \quad \vec{H} = \vec{B}/\mu$$

$$\vec{\nabla} \times (\vec{B}/\mu) = \frac{\epsilon}{c} \frac{\partial \vec{E}}{\partial t}$$

$$(d) \quad \vec{\nabla} \times \frac{1}{\mu} \frac{\partial \vec{B}}{\partial t} = \frac{\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$(e) \quad -\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$(f) \quad -\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) + \nabla^2 \vec{E} = \frac{\mu \epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$(g) \quad \vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{D}/\epsilon = P_{\text{free}}/\epsilon = 0$$

$$4.(h) \quad \vec{E} = \vec{E}(kx - wt)$$

$$\frac{\nabla^2 \vec{E}}{c^2} = k^2 \vec{E} \quad \text{cancel if}$$

$$\frac{\epsilon \mu}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{\epsilon \mu}{c^2} \omega^2 \vec{E} \quad \frac{\epsilon \mu}{c^2} \omega^2 = k^2$$

$$(i) \quad \frac{\omega}{k} = \frac{c}{\sqrt{\epsilon \mu}} \quad \text{from (h)}$$

5. Spherical symmetry \Rightarrow use Gauss' law (for \vec{E} or \vec{B}) with spherical shell statically, \vec{E} must be 0 within any conductor

$$\Rightarrow \boxed{\vec{E} = 0, r < a}$$

Place a shell at $r > b$

$$\oint \vec{D} \cdot d\vec{a} = 4\pi q_{\text{free}} \quad \hat{D} = \hat{r} \text{ by symmetry}$$

$$4\pi r^2 D_r = 4\pi q_0$$

$$\vec{D} = \hat{r} \frac{q_0}{r^2}$$

$$\boxed{\vec{D} = 0, r = \infty}$$

Place a shell at $a < r < b$

$$\vec{D} = \hat{r} \frac{q_0}{r^2} \text{ as above}$$

$$4\pi \vec{P} = \hat{r} \frac{2q_0}{(a+b)r}$$

$$\vec{E} = \vec{D} - 4\pi \vec{P} = \hat{r} \frac{q_0}{r^2} \left(1 - \frac{2r}{a+b}\right)$$

$$\boxed{\vec{E} = 0 \text{ when } r = \frac{a+b}{2}}$$

6.(a) Consider wall at constant positive x . In time Δt the # n of molecules hitting the wall is

$$n = \frac{NA}{2} v_x \Delta t \quad (N = \text{number density})$$

$\ell_p = \text{mass density}$

After elastic collision, wall receives momentum $2mv_x$:

$$\Delta p_x = (Nm) \frac{v_x}{2} \Delta v_x A \Delta t$$

$\ell_p = \text{mass density}$

$$\text{Pressure } p = \frac{F_x}{A} = \frac{\Delta p_x}{A \Delta t} = p \langle v_x^2 \rangle$$

$\ell_{\text{average over molecules}}$

$$\text{KE density } u = N \left(\frac{1}{2} m \langle v^2 \rangle \right) \quad (2)$$

$$= \frac{1}{2} p \langle v^2 \rangle$$

$$u = \frac{3}{2} p \langle v_x^2 \rangle$$

$$\ell = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

$$\frac{p}{u} = \frac{p \langle v_x^2 \rangle}{\frac{3}{2} p \langle v_x^2 \rangle} = \boxed{\frac{2}{3}}$$

(b.) Now the particles are fully relativistic, with $v = \hat{x}v_x + \hat{y}v_y + \hat{z}v_z$,

$$v_x^2 + v_y^2 + v_z^2 = c^2$$

$$\text{Again } n = \frac{NA}{2} v_x \Delta t.$$

$$\text{But } p_x = \frac{E}{c} \cdot \frac{v_x}{c} \quad E = \text{photon energy}$$

ℓ fraction in x direction
total momentum

Wall receives momentum

$$\Delta p_x = \left(\frac{NA}{2} v_x \Delta t \right) \left(\frac{1}{2} \frac{E}{c} \frac{v_x}{c} \right)$$

$$= (NE) \left(\frac{v_x^2}{c^2} \right) A \Delta t$$

ℓ energy density u

$$\text{With } \langle \frac{v_x^2}{c^2} \rangle = \frac{1}{3} \text{ this becomes}$$

$$\boxed{p = \frac{u}{3}}$$

$$(c.) \quad u = \frac{\langle E^2 + B^2 \rangle}{8\pi}$$

In an EM wave in vacuum $\langle E^2 \rangle = \langle B^2 \rangle$

$$\text{so } u = \frac{\langle B^2 \rangle}{4\pi}$$

$$\langle B^2 \rangle = 4\pi u = 12\pi p$$

$$\text{given } p = 10^6 \text{ dynes/cm}^2$$

$$\Rightarrow \langle B^2 \rangle^{1/2} \approx 6000 \text{ gauss}$$